A Real Option Case under Ambiguity

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1. Introduction

This paper selects an existing company and conducts a case study with respect to a new business for the company. We observe that the company is starting a membrane ceilings business alongside its existing business selling iron. We apply real options approach to actual business for valuing the business and providing valuable implications with regard to managerial decision-making under uncertainty. In addition to the standard option pricing theory under risk, the potential model risk is also analyzed. For this reason, we consider the value of the business under ambiguity on the optimal decision for the company. In order to analyze the actual business, we develop a systematic approach to analyzing managerial flexibility under uncertainty. This includes specifying important risk factors and corresponding real options, parameter estimation, handling the ambiguity, and deriving the optimal strategy. Our analysis reveals that the company has two real options under uncertainty with respect to market prices and demands. We show that these options have a significant impact on the project value of the membrane ceilings business and that the presence of model risk could change the optimal exercise timing of the real options.

2. Description of the case study

We first review the difficulties we faced in analyzing this as a real case study. First, the chief manager of the company initially did not know anything about real options, although he did have some basic knowledge about net present value (NPV) analysis. However, even if he has little idea about real options, he may still have possessed the necessary way of thinking about real options. Therefore, we interviewed him without using overly technical terminology to better grasp the present situation and help him identify the elements needed for our analysis. Second, for quantitative real option analysis, it is necessary to specify the underlying model, the flexibility that the company might have, and estimate the parameter values under limited information. In applying the real options approach to actual business, we also need to examine whether each theoretical assumption is valid. For example, we decided to utilize a geometric Brownian motion as an approximation of the underlying risk process, but we decided not to employ risk-neutral valuation as the no-arbitrage condition is not appropriate for the analysis. Third, even though we can decide which model to use, we do not know how to determine the parameters of the model when the company has no past data along with public data. One way for resolving it is to extract information from the manager's expectations. However, we need to recognize that these could entail bias and misspecification. To deal with this, we begin our analysis based on the manager's beliefs, and later examine it in the presence of ambiguity in response to possible misspecification by the manager. Finally, we must identify the real options, i.e., the managerial flexibility of the company. From a theoretical viewpoint, the company has many types of real options. However, in practice, they do not always existent or are important. Therefore, we need to identify existing real options that potentially affect the value of the new business. Fig.1 illustrates the four steps of analysis developed in this paper. The idea in the figure is based on the discussion in [2], but we extend it for our case study.

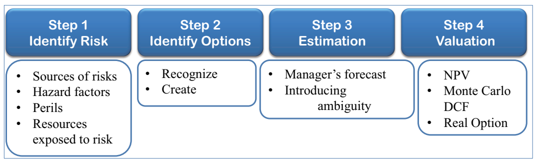

Figure 1: Four steps of analysis

In step 1, we have figured out important sources of risk that could affect the membrane business. After the series of intensive interviews with the chief manager of the company, we picked up the following four sources of uncertainty, i.e., area per unit order, price for the existing products per unit area, price for the value-added products per unit area, and ratio of the value added products to the total products. In step 2, although we have found several types of managerial flexibility as real options, we focus on two types of real options under the four sources of uncertainty for a practical application because we conclude they are the most critical factors for the quantitative analysis of the new business with real options. They are, to sell nonflammable products and to build a processing plant.

In Step 3, we discuss estimation of the parameters. When we apply the real options approach, we express risks as stochastic processes, for we need to estimate the parameters. However, there is no existing data

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available for membrane ceilings. Therefore, we employ the method of extracting the information from the manager's foresight. However, there is a possibility that the manager's prediction contains bias and misspecification. To deal with these, we evaluate the project value in the presence of ambiguity[1].

3. Valuation Model

In Step 4, we evaluate the new business and derive the optimal exercise strategy that maximizes the value of the new business. This paper employs NPV method, Monte Carlo DCF method, and real options approach to compare and analyze the new business. For the real option approach, we utilize trinomial lattice for modeling each underlying risk, hence, 3^4-nomial lattice for each period. To satisfy practical relevance we consider 6 years of project horizon with monthly decision opportunities. Note that the flexibility can be evaluated as a switching option with three stages.

Let $V_A(t,j)$ and $V_B(t,j), j = 0,1,2$ denote the project values after and before the decision making in stage j at time t, respectively. Using the dynamic programming principle, we can derive the following equations:

$$V_A(t,j) = e^{-r\Delta t}\mathbb{E}_t[V_B(t+1,j)],$$

where r is the instantaneous discount rate, Δt is the length of each time point and $\mathbb{E}_t[\cdot]$ represents the conditional expected value at time t, with

$$V_B(t,0) = CF(t,0) + \max\left(V_A(t,0), V_A(t,1) - C_{0,1}\right),$$

if the switching is possible where $CF(t,0)$ represents the cash-flow obtained in state 0 at time t, $C_{0,1}$ is a switching cost from stage 0 to stage 1. For valuing real options under ambiguity, we formulate it as a multiplier robust control problem, that is,

$$V_A^{\mathbb{Q}}(t,j) = \min_{\mathbb{Q}\in\mathcal{P}} e^{-r\Delta t}\left\{\mathbb{E}_t^{\mathbb{Q}}[V_B^{\mathbb{Q}}(t+1,j)] + \theta R_{t+1}(\mathbb{Q}|\mathbb{P})\right\},$$

$$V_B^{\mathbb{Q}}(t,0) = CF(t,0) + \max\left(V_A^{\mathbb{Q}}(t,0), V_A^{\mathbb{Q}}(t,1) - C_{0,1}\right).$$

In the equations, $R_{t+1}(\mathbb{Q}|\mathbb{P})$ represents Kullback-Leibler (KL) divergence, which measures the difference between two distributions.

4. Results

The benchmark result obtained by the NPV is summarized in Table 1. In this scenario, the company has no real option to exercise and simply continues the existing business. The NPV is around 290 million yen. We consider another scenario in which the company commences selling the value-added products

[1]See [1] and references therein.

Table 1: NPV of the first scenario

	2015	2016	2017	2018	2019	2020
Number of orders	3	6	15	30	45	60
Area per unit order (m^2)	16.95	19.80	23.12	27.01	31.54	36.84
Total area (m^2)	50.85	118.78	346.83	810.18	1419.39	2210.40
Price for the existing products per unit area (JPY)	39,835	37,639	35,564	33,603	31,750	30,000
Sales (JPY)	2,025,610	4,470,819	12,334,692	27,224,480	45,066,322	66,312,000
Profit rate (%)	50%	49%	48%	47%	46%	45%
Gross profit (JPY)	1,012,805	2,190,701	5,920,652	12,795,506	20,730,508	29,840,400
Labor cost (JPY)	7,000,000	7,500,000	8,000,000	8,500,000	9,000,000	9,500,000
Tax rate (%)	42%					
Capital expenditure (JPY)	1,000,000	1,500,000	2,000,000	2,500,000	3,000,000	3,500,000
CF (JPY)	-6,987,195	-6,809,299	-4,079,348	-8,607	3,803,695	8,297,432
Discount rate (%)	5%					
Expected growth rate (%)	3%					
Terminal value (JPY)						390,572,792
NPV (JPY)	294,666,889					

Table 2: NPV of the third scenario

	2015	2016	2017	2018	2019	2020
Number of orders	3	6	15	30	45	60
Area per unit order (m^2)	16.95	19.80	23.12	27.01	31.54	36.84
Total area (m^2)	50.85	118.78	346.83	810.18	1419.39	2210.40
Price for the existing products per unit area (JPY)	39,835	37,639	35,564	33,603	31,750	30,000
Price of the value-added products per unit area (JPY)		50,000	59,460	70,711	84,090	100,000
Ratio of the existing products to the total products (%)	100.00%	100.00%	95.76%	94.00%	91.51%	88.00%
Ratio of the value-added products to the total products (%)		(3.00%)	4.24%	6.00%	8.49%	(12.00%)
Percentage of demands of the value-added product (%)		(6.00%)	(8.49%)	(12.00%)	(16.97%)	24.00%
Sales (JPY)	2,025,610	4,470,819	12,686,329	29,028,309	51,370,020	111,404,160
Profit rate (%)	50%	49%	48%	47%	46%	45%
Gross profit (JPY)	1,012,805	2,190,701	6,089,438	13,643,305	23,630,209	50,131,872
Labor cost (JPY)	7,000,000	7,500,000	8,000,000	8,500,000	9,000,000	9,500,000
Tax rate (%)	-5,987,195	-5,309,299	-1,910,562	2,983,117	8,485,521	23,566,486
Capital expenditure (JPY)	1,000,000	1,500,000	3,000,000	2,500,000	3,000,000	83,500,000
CF (JPY)	-6,987,195	-6,809,299	-4,910,562	483,117	5,485,521	-59,933,514
Discount rate (%)	5%					
Expected growth rate (%)	3%					
Terminal value (JPY)						944,560,121
NPV (JPY)	671,669,607					

from 2017 and building the plant in 2020. The switching costs from Stage 0 to Stage 1 equals 1 million (JPY), and 80 million (JPY) from Stage 1 to Stage 2. Table 2 provides the results for the scenario. Note that it is the chief manager's predetermined strategy. We can see that although the CF for 2020 is negative because of the switching cost to stage 2, this investment increases the terminal value and makes the project more profitable than the second scenario. This clearly indicates the profitability of building the new plant.

Although the conditional exercise policy in scenario 3 is much more valuable than the the fixed scenario in scenario 1, it is not yet an optimal decision. With the real option approach we can derive the maximized project value under the optimal timing of switching. The maximized project value with the real options is 1,231 million yen, which is about 4.2 times more valuable as compared with the first scenario, and 1.8 times as compared with the chief manager's predetermined scenario.

Finally, we derive the project value in the presence of ambiguity, where the manager's misspecification is explicitly taken into account. We confirm that when the manager loses his confidence, he tends to choose smaller drift of the process, and the real options tend to be exercised later. Consequently, the project value is decreased under ambiguity.

References

[1] Imai, J. and Tsujimura, M, Assessing Capital Investment Strategy with Convex Adjustment Cost under Ambiguity, *under Review*, 2017.

[2] Trigeorgis, L. and Reuer, J., Real options theory in strategic management. *Strategic Management Journal*, **38**(1), 42–63 ,2002.